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2-Local higher derivations

Tayebeh Lal Shateri[a]

[a]*Faculty of Mathematics and Computer Science, Hakim Sabzevari University, Sabzevar, Iran*

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Abstract

The paper is devoted to 2-local higher derivations on some algebras. It is shown that continuous 2-local higher derivations on $\mathfrak{B}(\mathcal{H})$, for an infinite dimensional separable Hilbert space $\mathcal{H}$ are higher derivations and each 2-local inner higher derivation on $\mathfrak{F}(\mathcal{H})$ (the ideal of all finite-dimensional operators from $\mathfrak{B}(\mathcal{H})$) is a higher derivation. Also we prove that every 2-local higher derivation from a commutative $*$-subalgebra of the matrix algebra M_n over $\mathbb{C}$ is a higher derivation.

Keywords: Higher derivation, 2-local higher derivation, Hilbert space, Matrix algebra.
MSC 2000: Primary: 46H40; Secondary: 46H40.

1 introduction

Let $\mathcal{A}$ and $\mathcal{B}$ be algebras. A *higher derivation* of rank k (k might be ∞) is a family of linear mappings $\{d_m\}_{m=0}^k$ from $\mathcal{A}$ into $\mathcal{B}$ such that

$$d_m(ab) = \sum_{j=0}^{m} d_j(a)d_{m-j}(b), \quad (a, b \in \mathcal{A}, \quad m = 0, 1, 2, ...). \tag{1.1}$$

It is obvious that for a higher derivation $\{d_m\}_{m=0}^k$, d_0 is a homomorphism. Higher derivations were introduced by Hasse and Schmidt [5], and algebraists sometimes call them Hasse-Schmidt derivations. They are also studied in other contexts. In [17] higher derivations are applied to study generic solving of higher differential equations. The reader may find more about higher derivations in [2, 4, 6, 7, 9, 12, 13, 15, 16].
A higher derivation $\{d_m\}_{m=0}^k$ is said to be continuous if each d_m is continuous. If $\mathcal{B} = \mathcal{A}$ and $d_0 = id_{\mathcal{A}}$, where $id_{\mathcal{A}}$ is the identity map on $\mathcal{A}$, then d_1 is a derivation and $\{d_m\}_{m=0}^k$ is called a *strongly* higher derivation. A standard example of a higher derivation of rank k is $\{\frac{D^m}{m!}\}_{m=0}^k$ where $D : \mathcal{A} \to \mathcal{A}$ is a derivation. A strong higher derivation $\{d_m\}_{m=0}^k$ on $\mathcal{A}$ is called *inner* if there are $u_0, u_1, \cdots, u_k \in \mathcal{A}$ such that $d_m(a) = au_m - \sum_{i=1}^m u_{m-i}d_i(a)$, for all $a \in \mathcal{A}$ and $1 \le m \le k$. A family $\{d_m\}_{m=0}^k$ of linear mappings from an algebra $\mathcal{A}$ into an algebra $\mathcal{B}$ is called 2-*local higher derivation* (res. 2-*local inner higher derivation*) if for all $a, b \in \mathcal{A}$ there exists a higher derivation (res. an inner higher derivation) $\{D_m^{a,b}\}_{m=0}^k$ from $\mathcal{A}$ into $\mathcal{B}$ such that $d_m(a) = D_m^{a,b}(a)$ and $d_m(b) = D_m^{a,b}(b)$. In the context of derivations, the relation between 2-local derivations and derivations is studied by Several authors. Semrl in [14], shows that every 2-local derivation on $\mathfrak{B}(\mathcal{H})$, for an infinite dimensional separable Hilbert space $\mathcal{H}$ is derivation. Also

Email address: t.shateri@hsu.ac.ir (Tayebeh Lal Shateri)

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Og Kim and Seon Kim in [10, 11] proved that 2-local derivations on M_n and AF C^*-algebras are derivations. Ayupov and Kudaybergenov in [1] proved that every 2-local derivation on $\mathfrak{B}(\mathcal{H})$, for a Hilbert space $\mathcal{H}$, is a derivation.

In the present paper, we generalize some results of [14, 1, 10, 11] for 2-local higher derivations. We prove that continuous 2-local higher derivations on $\mathfrak{B}(\mathcal{H})$, for an infinite dimensional separable Hilbert space $\mathcal{H}$ and 2-local inner higher derivations on $\mathfrak{F}(\mathcal{H})$ is a higher derivation. Furthermore we show that every 2-local higher derivation from a commutative $*$-subalgebra of the matrix algebra M_n over $\mathbb{C}$ is a higher derivation.

2 2-local higher derivations

Before stating our results we fix some notation. Let $\mathcal{H}$ be a separable Hilbert space and let $\mathfrak{B}(\mathcal{H})$ be the algebra of all linear bounded operators on $\mathcal{H}$. We denote by $\mathfrak{F}(\mathcal{H})$, the ideal of all finite-dimensional operators from $\mathfrak{B}(\mathcal{H})$ and by tr the canonical trace on $\mathfrak{B}(\mathcal{H})$. For each $x, y \in \mathcal{H}$, xy^* denotes the rank one operator given by $(x \otimes y)z =< z, y > x \quad (z \in \mathcal{H})$. Also $id_{\mathcal{H}}$ denotes the identity operator on $\mathcal{H}$.

Theorem 2.1. Let $\mathcal{H}$ be an infinite dimensional separable Hilbert space and let $\{d_m\}_{m=0}^k$ be a 2-local strong higher derivation on $\mathfrak{B}(\mathcal{H})$ such that $d_m(id_{\mathcal{H}})$ commutes with $id_{\mathcal{H}}$ for all $1 \leq m \leq k$. Then $\{d_m\}_{m=0}^k$ is a higher derivation.

Proof . By employing the similar way of the proof of [14, Theorem 2], let $\{e_n\}$ be an orthonormal basis of $\mathcal{H}$. We define the operators α, β in $\mathfrak{B}(\mathcal{H})$ as follows

$$\alpha = \sum_{n=1}^{\infty} \frac{1}{2^n} e_n e_n^* , \quad \beta e_1 = 0 \quad \text{and} \quad \beta e_n = e_{n-1} , \quad (n \geq 2).$$

Now, since $\{d_m\}_{m=0}^k$ is a 2-local strong higher derivation on $\mathfrak{B}(\mathcal{H})$, hence there is a higher derivation $\{D_m^{\alpha,\beta}\}$ such that $d_m(\alpha) = D_m^{\alpha,\beta}(\alpha)$ and $d_m(\beta) = D_m^{\alpha,\beta}(\beta)$. By replacing d_m with $d_m - D_m^{\alpha,\beta}$ we can assume that $d_m(\alpha) = d_m(\beta) = 0$. Hence it is enough to prove that $d_m(T) = 0$, for all $T \in \mathfrak{B}(\mathcal{H})$ and for all $1 \leq m \leq k$. If $k = 1$, the proof of [14, Theorem 2] shows that $d_1(T) = 0$, for all $T \in \mathfrak{B}(\mathcal{H})$. Now, assume that $d_m(T) = 0$, for all $T \in \mathfrak{B}(\mathcal{H})$ and for all $1 \leq m \leq k-1$, we show that $d_k(T) = 0$. Suppose $S, T \in \mathfrak{B}(\mathcal{H})$, since $\{d_m\}_{m=0}^k$ is a 2-local strong higher derivation, so there is a higher derivation $\{D_m^{S,T}\}_{m=0}^k$ such that $d_m(S) = D_m^{S,T}(S)$ and $d_m(T) = D_m^{S,T}(T)$ for all $1 \leq m \leq k-1$. On the other hand each 2-local higher derivation is a local higher derivation, hence there exists a higher derivation $\{D_m^{ST}\}_{m=0}^k$ on $\mathfrak{B}(\mathcal{H})$ such that $d_m(ST) = D_m^{ST}(ST)$ for all $1 \leq m \leq k-1$. We have

$$\begin{aligned} D_k^{S,T}(ST) &= SD_k^{S,T}(T) + \sum_{m=1}^{k-1} D_m^{S,T}(S) D_{k-m}^{S,T}(T) + D_k^{S,T}(S)T \\ &= Sd_k(T) + \sum_{m=1}^{k-1} d_m(S) d_{k-m}(T) + d_k(S)T = Sd_k(T) + d_k(S)T. \end{aligned}$$

Therefore $D_k^{ST}(ST) - D_k^{S,T}(ST) = d_k(ST) - Sd_k(T) + d_k(S)T$. Now by replacing S with $id_{\mathcal{H}}$, we get

$$D_k^{id_{\mathcal{H}}T}(T) - D_k^{id_{\mathcal{H}},T}(T) = d_k(T) - d_k(T) + d_k(id_{\mathcal{H}})T.$$

□

With the help of [1, Lemma 2.2, Theorem 2.3], the following result can be derived for a 2-local strongly higher derivation.

Lemma 2.2. Let $\{d_m\}_{m=0}^k$ be a 2-local strongly higher derivation on $\mathfrak{B}(\mathcal{H})$ such that $d_m|_{\mathfrak{F}(\mathcal{H})} = 0$, for all $1 \leq m \leq k$. Then $d_m = 0$, for all $1 \leq m \leq k$.

Proof . For each $S, T \in \mathfrak{B}(\mathcal{H})$ there exists a strongly higher derivation $\{D_m^{S,T}\}_{m=0}^k$ such that $d_m(S) = D_m^{S,T}(S)$ and $d_m(T) = D_m^{S,T}(T)$ for all $1 \leq m \leq k$. By [1, Theorem 2.3], d_1 is a derivation. But by [1, Lemma 2.2] we know that $d_1 = 0$ so $D_1^{x,y}(x) = D_1^{x,y}(y) = 0$. Hence d_2 is a 2-local derivation and [1, Lemma 2.2] implies that $d_2 = 0$. Continuing this process we conclude that $d_m = 0$, for $1 \leq m \leq k$. □

In the following theorem, we consider 2-local strongly inner higher derivations on $\mathfrak{B}(\mathcal{H})$, for a separable Hilbert space $\mathcal{H}$.

Theorem 2.3. Let $\mathcal{H}$ be a separable Hilbert space and let $\{d_m\}_{m=0}^k$ be a 2-local strongly inner higher derivation on $\mathfrak{B}(\mathcal{H})$. Then the restriction of $\{d_m\}_{m=0}^k$ on $\mathfrak{F}(\mathcal{H})$ is a higher derivation.

Proof . Let $S,T \in \mathfrak{B}(\mathcal{H})$, then there exists a strongly inner higher derivation $\{D_m^{S,T}\}_{m=0}^k$ such that $d_m(S) = D_m^{S,T}(S)$ and $d_m(T) = D_m^{S,T}(T)$ for all $1 \leq m \leq k$. Since $\{D_m^{S,T}\}_{m=0}^k$ is inner there exist $u_0 = 1, u_1, \ldots, u_m$ in $\mathfrak{B}(\mathcal{H})$ such that

$$D_m^{S,T}(ST) = STu_m - u_mST - \sum_{i=1}^{m-1} u_{m-i}D_i^{S,T}(ST).$$

But

$$D_m^{S,T}(ST) = \sum_{i=1}^{m} D_{m-i}^{S,T}(S)D_i^{S,T}(T) = \sum_{i=1}^{m} d_{m-i}(S)d_i(T).$$

Hence

$$STu_m - u_mST - \sum_{i=1}^{m-1} u_{m-i}D_i^{S,T}(ST) = \sum_{i=1}^{m} d_{m-i}(S)d_i(T).$$

[1, Theorem 2.3] implies that d_1 is a derivation. Now let $S,T \in \mathfrak{F}(\mathcal{H})$, since the trace tr accepts finite values on $\mathfrak{F}(\mathcal{H})$ and $\mathfrak{F}(\mathcal{H})$ is an ideal in $\mathfrak{B}(\mathcal{H})$ we have

$$tr(d_1(ST)) = tr((d_1S)T) = tr(T(d_1S)) = tr((Td_1)S) = tr(S(Td_1)) = tr((ST)d_1),$$

hence $tr(d_1(ST)) = 0$. Similarly it is deduced that

$$tr(\sum_{i=0}^{m} d_{m-i}(S)d_i(T)) = tr(D_m^{S,T}(ST)) = 0,$$

for $1 \leq m \leq k$. Hence

$$tr(Sd_m(T)) = -tr(d_m(S)T) - tr(\sum_{i=1}^{m-1} d_{m-i}(S)d_i(T)). \tag{2.1}$$

Now for $V,W,Z \in \mathfrak{F}(\mathcal{H})$, set $X = V + W,\ T = Z$, then we have

$$\begin{aligned} tr((V+W)d_m(Z)) &= -tr(d_m(V+W)Z) - \sum_{i=1}^{m-1} tr(d_{m-i}(V+W)d_i(Z)) \\ &= -tr(d_m(V+W)Z) - \sum_{i=1}^{m-1} [tr(d_{m-i}(V)d_i(Z)) + tr(d_{m-i}(W)d_i(Z))], \end{aligned}$$

and so

$$tr(Vd_m(Z)) + tr(Wd_m(Z)) = -tr(d_m(V+W)Z) - \sum_{i=1}^{m-1} [tr(d_{m-i}(V)d_i(Z)) + tr(d_{m-i}(W)d_i(Z))].$$

By (2.1) we obtain

$$\begin{aligned} tr(Vd_m(Z)) + tr(Wd_m(Z) &= -tr(d_m(V)Z) - \sum_{i=1}^{m-1} [tr(d_{m-i}(V)d_i(Z)) \\ &\quad - tr(d_m(W)Z) - \sum_{i=1}^{m-1} [tr(d_{m-i}(W)d_i(Z)) \\ &= -tr(d_m(V+W)Z) - \sum_{i=1}^{m-1} [tr(d_{m-i}(V)d_i(Z)) + tr(d_{m-i}(W)d_i(Z))], \end{aligned}$$

hence $tr([d_m(V+W)-d_m(V)-d_m(W)]Z)=0$ for all $V,W,Z\in\mathfrak{F}(\mathcal{H})$. Now if $a=d_m(V+W)-d_m(V)-d_m(W)$ and $Z=A^*$, then $tr(AA^*)=0$, since the trace tr is faithful, it follows that $AA^*=0$ and so $A=0$. Therefore

$$d_m(V+W)=d_m(V)+d_m(W),$$

that is d_m is an additive map on $\mathfrak{F}(\mathcal{H})$. Now let $S\in\mathfrak{B}(\mathcal{H})$ and $\lambda\in\mathbb{C}$, there exists an inner higher derivation $\{D_m^{S,\lambda S}\}_{m=0}^k$ such that $d_m(S)=D_m^{S,\lambda S}(S)$ and $d_m(\lambda S)=D_m^{S,\lambda S}(\lambda S)$ for $1\le m\le k$, then for $1\le m\le k$

$$d_m(\lambda S)=D_m^{S,\lambda S}(\lambda S)=\lambda D_m^{S,\lambda S}(S)=\lambda d_m(S).$$

Consequently d_m is a linear operator. Now, for each $S\in\mathfrak{B}(\mathcal{H})$ there exists an inner higher derivation $\{D_m^{S,S^2}\}_{m=0}^k$ such that $d_m(S)=D_m^{S,S^2}(S)$ and $d_m(S^2)=D_m^{S,S^2}(S^2)$ for $1\le m\le k$. Then

$$d_m(S^2)=D_m^{S,S^2}(S^2)=\sum_{i=0}^{m}D_{m-i}^{S,S^2}(S)D_i^{S,S^2}(S)=\sum_{i=0}^{m}d_{m-i}(S)d_i(S).$$

Therefore, the restriction $\{d_m\}_{m=0}^k|_{\mathfrak{F}(\mathcal{H})}$ is a linear Jordan higher derivation on $\mathfrak{F}(\mathcal{H})$. Since $\mathfrak{F}(\mathcal{H})$ is semi-prime, [3, Theorem 1.2] implies that $\{d_m\}_{m=0}^k$ on $\mathfrak{F}(\mathcal{H})$ is a higher derivation on $\mathfrak{F}(\mathcal{H})$. This completes the proof. □

Now we consider 2-local higher derivations on M_n. For this we need the following theorem which is proved in [8, Theorem 3.5].

Theorem 2.4. Let $\mathfrak{M}$ be a W^*-algebra and $\{d_m\}_{m=0}^k$ a strongly higher derivation on $\mathfrak{M}$. Let $\mathcal{A}$ be a commutative W^*-subalgebra of $\mathfrak{M}$. Then for each $m\in\mathbb{N}$ there are $u_0=1,u_1,\ldots,u_m$ in $\mathfrak{M}$ such that $d_m(a)=au_m-\sum_{i=0}^{m-1}u_{m-i}d_i(a)$ for all $a\in\mathcal{A}$ and

$$\|u_m\|\le\|d_m\|+\|u_{m-1}\|\|d_1\|+\ldots+\|u_1\|\|d_{m-1}\|.$$

Theorem 2.5. Let M_n be an $n\times n$ matrix algebra over $\mathbb{C}$. Then every 2-local higher derivation from a commutative $*$-subalgebra of M_n into M_n is a higher derivation.

Proof . Let $\{d_m\}_{m=0}^k$ be a local higher derivation on a commutative $*$-subalgebra $\mathcal{N}$ of M_n and let $\{e_1,e_2,\cdots,e_n\}$ be the standard basis for $\mathbb{C}^n$. We consider two matrices A and B in M_n as follows

$$A=\begin{pmatrix}\frac{1}{2} & 0 & \cdots & 0\\ 0 & \frac{1}{2^2} & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots\\ 0 & \cdots & 0 & \frac{1}{2^n}\end{pmatrix},\quad B=\begin{pmatrix}0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 0 & 1\\ 0 & 0 & \cdots & 0 & 0\end{pmatrix}.$$

It is easy to see that $T\in M_n$ commutes with A if and only if it is diagonal, and $U\in M_n$ commutes with B if U is of the form

$$Ue_k=\sum_{i=1}^{k}\lambda_{k+1-i}e_i\quad(k=1,2,\cdots,n)$$

for some $\{\lambda_1,\cdots,\lambda_k|\quad\lambda_k\in\mathbb{C}\}$.
By assumption there is a higher derivation $\{D_m^{A,B}\}$ on M_n such that $d_m(A)=D_m^{A,B}(A),\ d_m(B)=D_m^{A,B}(B)\quad(m\in\mathbb{N})$. Replacing d_m by $d_m-D_m^{A,B}$, if necessary, we can assume that $d_m(A)=d_m(B)=0$. It is enough to show that for each $T\in M_n$, $d_m(T)=0$. By [10, Theorem 3], it follows that $d_1(T)=0$. Suppose that $d_i=0$ $(1\le i\le m-1)$. Since $D_m^{A,B}$ is a higher derivation Theorem (2.4) implies that for any $T\in M_n$, there exist diagonal elements $\{V_m^{A,T}\}$ and $\{U_m^{B,T}\}$ of the above form, depending on T, such that

$$D_m^{A,T}(T)=TV_m^{A,T}-V_m^{A,T}T-\sum_{k=1}^{m-1}V_k^{A,T}D_{m-k}^{A,T}(T)=TV_m^{A,T}-V_m^{A,T}T,$$

$$D_m^{B,T}(T)=TU_m^{B,T}-U_m^{B,T}T-\sum_{k=1}^{m-1}U_k^{B,T}D_{m-k}^{B,T}(T)=TU_m^{B,T}-U_m^{B,T}T.$$

But $D_m^{A,T}(T) = d_m(T) = D_m^{B,T}(T)$, hence $TV_m^{A,T} - V_m^{A,T}T = TU_m^{B,T} - U_m^{B,T}T$. Let $\{I_{ij}\}_{i,j=1,\cdots,n}$ be the system of matrix units of M_n. Then for any i and j we have

$$d_m(I_{ij}) = I_{ij}V_m^{A,I_{ij}} - V_m^{A,I_{ij}}I_{ij} = I_{ij}U_m^{B,I_{ij}} - U_m^{B,I_{ij}}I_{ij},$$

for some $V_m^{A,I_{ij}} = diag(\lambda_{1m}, \cdots, \lambda_{nm})$ and $U_m^{B,I_{ij}}$ of the above form. Since

$$I_{ij}V_m^{A,I_{ij}} - V_m^{A,I_{ij}}I_{ij} = (\lambda_{jm} - \lambda_{im})I_{ij},$$

and (i,j)-entry of $I_{ij}U_m^{B,I_{ij}} - U_m^{B,I_{ij}}I_{ij}$ is zero, it follows that $d_m(I_{ij}) = 0$. But I_{ij} is the rank one operator $e_i \otimes e_j$, for any $T \in M_n$ we have

$$I_{ij}d_m(T)I_{ij} = D_m^{I_{ij},T}(I_{ij}TI_{ij}) =< Te_i, e_j > D_m^{I_{ij},T}(I_{ij}) =< Te_i, e_j > d_m(I_{ij}) = 0.$$

Consequently $< d_m(T)e_i, e_j > (I_{ij}) = 0$ and hence $d_m(T) = 0$, completing the proof. □

Employing the similar way as in [11, Prposition 3], we get the next corollary for a 2-local strongly higher derivation.

Corollary 2.6. Let $\mathcal{A} = M_1 \oplus M_2 \oplus \cdots \oplus M_n$, where M_i is a $k(i) \times k(i)$ matrix algebra for some $k(i) \in \mathbb{N}$. If $\{d_m\}_{m=0}^k$ is a 2-local strongly higher derivation from a commutative $*$-subalgebra $\mathcal{B}$ of $\mathcal{A}$ into $\mathcal{A}$, then $\{d_m\}_{m=0}^k$ is a higher derivation.

Proof . By [11, Prposition 3] it follows that d_1 is a derivation. Suppose that $A, B \in \mathcal{A}$, then there exists a higher derivation $\{D_m^{A,B}\}$ on $\mathcal{A}$ such that $d_m(A) = D_m^{A,B}(A)$ and $d_m(B) = D_m^{A,B}(B)$. Theorem (2.4) implies that for every $A, B \in \mathcal{A}$ there exist $U_0 = 1, U_1^{A,B}, \ldots, U_m^{A,B}$ in $\mathcal{A}$ such that

$$d_m(A) = AU_m^{A,B} - U_m^{A,B}A - \sum_{k=1}^{m-1} U_{m-k}^{A,B}D_k^{A,B}(A),$$
$$d_m(B) = BU_m^{A,B} - U_m^{A,B}B - \sum_{k=1}^{m-1} U_{m-k}^{A,B}D_k^{A,B}(B).$$

Suppose that $d_m^i = d_m|_{M_i}$, then $\{d_m^i\}$ is a 2-local higher derivation of M_i. By Theorem 2.5, $\{d_m^i\}$ is a higher derivation and hence it is inner. Let

$$d_m^i(A_i) = A_iS_m^i - S_m^iA_i - \sum_{k=1}^{m-1} S_{m-k}^i d_k^i(A_i) \quad A_i, S_1^i, \cdots S_k^i \in M_i \quad (1 \le k \le m).$$

Let $A = (A_1, \cdots, A_n)$ and $U_m^{A,B} = (U_1^m, \cdots, U_n^m)$, Since

$$\begin{aligned} d_m(A_1, \cdots, A_n) &= (A_1, \cdots, A_n)(U_1^m, \cdots, U_n^m) - (U_1^m, \cdots, U_n^m)(A_1, \cdots, A_n) \\ &- \sum_{k=1}^{m-1}(U_1^{m-k}, \cdots, U_n^{m-k})D_k^{A,B}(A_1, \cdots, A_n) \\ &= d_m^1(A_1) \oplus \cdots \oplus d_m^n(A_n) \\ &= \bigoplus_{i=1}^{n}\big(A_iS_m^i - S_m^iA_i - \sum_{k=1}^{m-1} S_{m-k}^i d_k^i(A_i)\big) \\ &= (A_1, \cdots, A_n)(S_1^m, \cdots, S_n^m) - (S_1^m, \cdots, S_n^m)(A_1, \cdots, A_n) \\ &- \sum_{k=1}^{m-1}(S_1^{m-k}, \cdots, S_n^{m-k})D_k^{A,B}(A_1, \cdots, A_n), \end{aligned}$$

it follows that $\{d_m\}_{m=0}^k$ is a higher derivation. □

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